Advanced Operations
Management
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Homework 5 Solution

P. 141 #1

Q1

G = gain by ordering one more, if needed

$= \$12000 - \$10,000$

$\$10,000$: cost if order now

$\$12000$: Cost if reorder

$= \$2000$

L = loss by ordering one more if not needed

$= \$10,000 - \$9,000$

$\$9,000$: End of year sale

$\$10,000$: Cost if ordered now

$= \$1,000$

$$\text{Critical Fractile} = \frac{G}{G+L} = \frac{2000}{2000+1000} = \frac{2}{3}$$

Demand Estimate

d	$P\{D=d\}$	$P\{D\le d\}$	
20	.3	.3	
25	.15	.45	
30	.15	.60	
35	.20	.80	← first value $\ge 2/3$
40	.20	1.00	

$\Rightarrow$ $\boxed{\text{order 35 cars}}$

$\boxed{\text{Q2}}$ $\boxed{\text{P. 142 \#6}}$

G = gain from "stocking" one more, if needed

= \$54 − \$30

\$54: Cost to have regular teacher fill in

\$30: Cost of substitute in pool

= \$24

L = loss from "stocking" one more, if <u>not</u> needed

= \$30

$$cf = \frac{G}{G+L} = \frac{24}{24+30} = \frac{24}{54} = \frac{4}{9} \approx .444$$

d	P{d = D}	P{D ≤ d}
200	.03	.03
275	.03	.06
350	.03	.09
400	.05	.14
450	.40	.54 ← First ≥ .444
⋮	⋮	⋮

$\boxed{\text{Put 450 substitute teachers in the pool}}$

Q3 Confederate Pulp and Paper

I assume the mill operates continuously, as stated on pages 17 and 18.

[redacted]

The variable cost of cutting and transporting a cunit of wood is

\$23	variable cutting cost
\$ 8	transport
\$ 2	handling
\$33	

The "full cost" of \$47.50 includes fixed costs, and is thus meaningless for microeconomic decision making.

Now we can compute

G = gain from stocking one extra cunit, if needed

= \$65 − \$33 = \$32

\$65: buying wood locally

\$33: our own cost

Now, let's consider L. If a log is not needed, it will sit on the pile until next Fall. That means we paid the following costs earlier than we needed to:

Variable cutting:	\$23.00
Variable transport:	\$ 8.00
Handling	$\boxed{\$\ 2.00}$?

The issue here is that it's not clear how the handling breaks down between putting the log on the pile — which we'd be doing a year earlier than necessary — and taking it off. But clearly the total cost incurred earlier than needed is between \$31 = \$23 + \$8 and \$33 = \$23 + \$8 + \$2.

Our cost of funds is 20% per year, so L is between (.2)(31) = \$6.20 and (.2)(33) = \$6.60.

So G/G+L is between 32/(32 + 6.20) = 0.838 and 32/(32+ 6.20) = .829

Now let's look at the demand distribution. Exhibit 2 seems more reliable than Exhibit 1 because

- It has more data
- It's not absolutely clear that in the past the mill operated at 100% capacity through the winter as it's supposed to this winter

The spreadsheet on the next page shows that with either p = .838 or p = .829, we get that we should plan for 159 freeze days, which we multiply by

	A	B	C	D	E	F	G	H
1	**Confederated Pulp and Paper Case**					**Consumption/Week**	4.8	**x 1000 cunit**
2						**Consumption/Day**	0.686	**x 1000 cunit**
3			**1000's**					
4		**Freeze**	**Cunits**	**Cumulative**			**Log**	
5	**Year**	**Days**	**Wood**	**Probability**			**Cost**	
6	1980	120	82.3	0.1		**Variable Cutting**	$ 23.00	
7	1987	130	89.1	0.2		**Variable Transport**	$ 8.00	
8	1984	138	94.6	0.3		**Handling**	$ 2.00	
9	1978	142	97.4	0.4		**Total Variable Cost**	$ 33.00	
10	1982	144	98.7	0.5				
11	1985	146	100.1	0.6		**Funds Rate**	20%	
12	1981	148	101.5	0.7				
13	1979	151	103.5	0.8		**Cost of Local Wood**	$ 65.00	
14	1986	159	109.0	0.9				
15	1983	170	116.6	1		**G =**	$ 32.00	
16						**L, lower estimate =**	$ 6.20	
17						**L, upper estimate =**	$ 6.60	
18		Sorted by freeze days						
19				**Critical Fractile, low L estimate**			0.838	
20				**Critical Fractile, high L estimate**			0.829	

$$4800 \; \frac{\text{units}}{\text{week}} \div 7 \; \frac{\text{days}}{\text{week}} \approx 686 \; \frac{\text{units}}{\text{day}}$$

to obtain about $\boxed{\text{109,000 units.}}$

Note that the critical fractile level of about .83 indicates that running out of wood about one year out of six is the right strategy from an EMV point of view. That is precisely what is happening in the data in Exhibit 1 (although perhaps not intentionally).