Advanced Operations -- Homework 7 Solutions

Q1

Optimal cost is 703.2
Production amounts: 14 0 0 15 0 7

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<h1>Advanced Operations -- Homework 7 Solutions</h1>

<h2>Q1</h2>

<script type="text/javascript">

var hugeNumber = 1e20

var stages            = 6
var startInventory    = 3
var inventoryCapacity  = 15
var productionCapacity = 15

var setupCost    = new Array(0,120,125,130,125,120,110)
var variableCost = new Array(0,8,10,9.5,9.5,10,10)
var holdingCost  = new Array(0,0.5,0.6,0.6,0.7,0.75,0.7)
var demand       = new Array(0,3,4,5,9,11,7)

var f = new Array()
var x = new Array()

for(t=1; t<=stages; t++)
{
  f[t] = new Array()
  x[t] = new Array()
}
f[stages+1] = new Array()

for(i=0; i<=inventoryCapacity; i++)
{
  f[stages+1][i] = 0
}

for(t=stages; t>0; t--)
{
  for(i=0; i<=inventoryCapacity; i++)
  {
    minProduction = demand[t] - i
    if (minProduction < 0)
      { minProduction = 0 }
    maxProduction = inventoryCapacity - i + demand[t]
    if (maxProduction > productionCapacity)
      { maxProduction = productionCapacity }

    value = hugeNumber
    bestMove = 0

    for(p=minProduction; p<=maxProduction; p++)
    {
       j = i + p - demand[t]
       productionCost = 0.0
       if (p > 0)
         { productionCost = setupCost[t] + variableCost[t]*p }
       moveValue = holdingCost[t]*j + productionCost + f[t+1][j]
       if (moveValue < value)
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        {
          value = moveValue
          bestMove = p
        }
     }

     f[t][i] = value
     x[t][i] = bestMove
   }
}

document.write("Optimal cost is " + f[1][startInventory] + "<br />")

document.write("Production amounts:")
i = startInventory
for(t=1; t<=stages; t++)
{
  document.write(" " + x[t][i])
   i = i + x[t][i] - demand[t]
}
document.write("<br />")


</script>

</body></html>
```

Q2

I saw a huge variety of solutions to this problem, including integer programming Solver models, and enumeration of all possible solutions. Note that enumeration of all possible solutions will be much slower than dynamic programming for larger models, especially ones with more precincts. Here is the output of a JavaScript solution; the program follows, along with a manual solution. Note that the car optimum car allocation is not unique; there are in fact 5 ways to acheive the minimum value of 37.

Minimum number of crimes is 37
Car allocations: 1 1 3

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<h2>Q2</h2>

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programming Solver models, and enumeration of all possible solutions.
Note that enumeration of all possible solutions will be much slower
than dynamic programming for larger models, especially ones with more
precincts.

Here is the output of a JavaScript solution; the program follows,
along with a manual solution.  Note that the car optimum car
allocation is not unique; there are in fact 5 ways to acheive the
minimum value of 37.

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<br>
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<script type="text/javascript">

var hugeNumber = 1e20

var stages   = 3
var cars     = 5

var crimes = new Array()
crimes[1] = new Array(14,10,7,4,1,0)
crimes[2] = new Array(25,19,16,14,12,11)
crimes[3] = new Array(20,14,11,8,6,5)

var f = new Array()
var x = new Array()

for(t=1; t<=stages; t++)
{
  f[t] = new Array()
  x[t] = new Array()
}
f[stages+1] = new Array()

for(i=0; i<=cars; i++)
{
  f[stages+1][i] = 0
}

for(t=stages; t>0; t--)
{
  for(i=0; i<=cars; i++)
  {
    value = hugeNumber
    bestMove = 0

    for(tryx=0; tryx<=i; tryx++)
    {
       moveValue = crimes[t][tryx] + f[t+1][i - tryx]
       if (moveValue < value)
       {
```

```
          value = moveValue
          bestMove = tryx
        }
    }
    f[t][i] = value
    x[t][i] = bestMove
  }
}

document.write("Minimum number of crimes is " + f[1][cars] + "<br>")

document.write("Car allocations:")
i = cars
for(t=1; t<=stages; t++)
{
  document.write(" " + x[t][i])
  i = i - x[t][i]
}
document.write("<br>")


</script>

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```

Here is a manual solution to Q2 (apologies for the formatting, but it should be understandable)

4. Define $f_t(i)$ to be the minimum number of crimes committed in precincts t, $t + 1$,...3 if i squad cars may be allocated to these precincts. Also define $x_t(i)$ to be the number of cars that should be assigned to precinct t in order to attain $f_t(i)$.

If we enter the last stage with c cars, we should deploy them all there. Thus,

$f_3(0) = 20$ $x_3(0) = 0$
$f_3(1) = 14$ $x_3(1) = 1$
$f_3(2) = 11$ $x_3(2) = 2$
$f_3(3) = 8$ $x_3(3) = 3$
$f_3(4) = 6$ $x_3(4) = 4$
$f_3(5) = 5$ $x_3(5) = 5$

Then

$f_2(0) = 25 + f_3(0) = 45^*$ (Assign 0 to Pre. 2) $x_2(0) = 0$

$$f_2(1) = \min \begin{cases} 25 + f_3(1) = 39^* & \text{(Assign 0 Cars to Pre. 2)} \\ & x_2(1) = 0 \text{ or } 1 \\ 19 + f_3(0) = 39^* & \text{Assign 1 Car to Pre. 2)} \end{cases}$$

$$f_2(2) = \min \begin{cases} 25 + f_3(2) = 36 & \text{(Assign 0 cars to Pre. 2)} \\ 19 + f_3(1) = 33^* & \text{(Assign 1 Car to Pre. 2)} \quad x_2(2) = 1 \\ 16 + f_3(0) = 36 & \text{(Assign 2 Cars to Pre. 2)} \end{cases}$$

$$f_2(3) = \min \begin{cases} 25 + f_3(3) = 33 & \text{(Assign 0 Cars to Pre. 2)} \\ 19 + f_3(2) = 30^* & \text{(Assign 1 Car to Pre. 2)} \quad x_2(3) = 1 \text{ or } 2 \\ 16 + f_3(1) = 30^* & \text{(Assign 2 Cars to Pre. 2)} \\ 14 + f_3(0) = 34 & \text{(Assign 3 Cars to Pre. 2)} \end{cases}$$

$$f_2(4) = \min \begin{cases} 25 + f_3(4) = 31 & \text{(Assign 0 Cars to Pre. 2)} \\ 19 + f_3(3) = 27^* & \text{(Assign 1 Car to Pre. 2)} \quad x_2(4) = 1 \text{ or } 2 \\ 16 + f_3(2) = 27^* & \text{(Assign 2 Cars to Pre. 2)} \\ 14 + f_3(1) = 28 & \text{(Assign 3 Cars to Pre. 2)} \\ 12 + f_3(0) = 32 & \text{(Assign 4 Cars to Pre. 2)} \end{cases}$$

$$f_2(5) = \min \begin{cases} 25 + f_3(5) = 30 & \text{(Assign 0 Cars to Pre. 2)} \\ 19 + f_3(4) = 25 & \text{(Assign 1 Car to Pre. 2)} \\ 16 + f_3(3) = 24^* & \text{(Assign 2 Cars to Pre. 2)} \quad x_2(5) = 2 \\ 14 + f_3(2) = 25 & \text{(Assign 3 Cars to Pre. 2)} \\ 12 + f_3(1) = 26 & \text{(Assign 4 Cars to Pre. 2)} \\ 11 + f_3(0) = 31 & \text{(Assign 5 Cars to Pre. 2)} \end{cases}$$

$$f_1(5) = \min \begin{cases} 14 + f_2(5) = 38 \text{ (Assign 0 Cars to Pre. 1)} \\ 10 + f_2(4) = 37^* \text{(Assign 1 Car to Pre. 1)} \\ 7 + f_2(3) = 37^* \text{(Assign 2 Cars to Pre. 1)} \\ 4 + f_2(2) = 37^* \text{(Assign 3 Cars to Pre. 1)} \\ 1 + f_2(1) = 40 \text{ (Assign 4 Cars to Pre. 1)} \\ 0 + f_2(0) = 45 \text{ (Assign 5 Cars to Pre. 1)} \end{cases}$$

$$x_1(5) = 1,2 \text{ or } 3$$

Tracing through all the possible values of the $x_t(i)$ starting with $x_3(5)$, we obtain a complete listing of optimal solutions:

Precinct 1	Precinct 2	Precinct 3	Total Crimes
3	1	1	37
2	2	1	37
2	1	2	37
1	2	2	37
1	1	3	37

Q3 Here is a manual solution to Q3 (apologies for the formatting, but it should be understandable)

3\.

Number of Years Phone is Kept before Trade-in	Net Cost of Operation
1	40 + 20 = $60
2	40 + 20 + 30 = $90
3	40 + 20 + 30 + 40 = $130
4	40 + 20 + 30 + 40 + 60 = $190
5	40 + 20 + 30 + 40 + 60 + 70 = $260

Let g(6) = 0 and define for t≤5, g(t) = minimum cost of maintaining a phone from time t to end of year 5 given that we have a new phone at time t(g(t) includes the cost of purchasing the phone that was purchased at time t). Then

g(5) = 60* (Keep phone for 1 year)

g(4) = min {
60 + g(5) = 120 (Keep phone for 1 year)
90 + g(6) = 90* (Keep phone for 2 years)

g(3) = min {
60 + g(4) = 150 (Keep phone for 1 year)
90 + g(5) = 150 (Keep phone for 2 years)
130 + g(6) = 130* (Keep phone for 3 years)

g(2) = min {
60 + g(3) = 190 (Keep phone for 1 year)
90 + g(4) = 180* (Keep phone for 2 years)
130 + g(5) = 190 (Keep phone for 3 years)
190 + g(6) = 190 (Keep phone for 4 years)

g(1) = min {
60 + g(2) = 240 (Keep phone for 1 year)
90 + g(3) = 220* (Keep phone for 2 years)
130 + g(4) = 220* (Keep phone for 3 years)
190 + g(5) = 250 (Keep phone for 4 years)
260 + g(6) = 260 (Keep phone for 5 years)

g(0) = min {
60 + g(1) = 280 (Keep phone for 1 year)
90 + g(2) = 270 (Keep phone for 2 years)
130 + g(3) = 260* (Keep phone for 3 years)
190 + g(4) = 280 (Keep phone for 4 years)
260 + g(5) = 320 (Keep phone for 5 years)

Thus, a phone purchased at time 0 should be kept for three years. Then another phone should be purchased and kept for three years. Total cost incurred is $260.

Q4 Define $f_t(w)$ to be the maximum net profit (revenues less costs) obtained from the steer during weeks t, t + 1,...,10, given that the steer weighs w pounds at the beginning of week t. Let $P(w)$ denote the amounts of feed it is possible to give a steer that already weighs w pounds. The key to the problem is to remember that revenues are only earned during week 10 (because the steer is only sold once!). Then

$$f_{10}(w) = \max_{p \in P(w)} \{10g(w, p) - c(p)\}.$$

Then, for $t \leq 9$,

$$f_t(w) = \max_{p \in P(w)} \{-c(p) + f_{t+1}(g(w, p))\}.$$

Farmer Jones should then work backwards until he computes $f_1(w_0)$.