Business Decision Analytics under Uncertainty
Spring 2018, Professor Eckstein
Homework 2

Due Wednesday, February 14

Q1: Gas Drilling

Your firm is in the natural gas extraction business, and has acquired rights to a plot of land which it believes has a 50% chance of containing $40 million worth of natural gas, and a 50% chance of being completely "dry" (that is, containing no commercially exploitable gas). Drilling a full-scale gas well on the site will cost $21 million.

Before deciding whether to drill, you have the option to perform a "test bore", which costs only $2 million. The "test bore" can have three possible results, "positive", "inconclusive", and "negative". For past sites that have contained commercially exploitable gas, your test bore procedure has yielded a positive result 50% of the time, an inconclusive result 30% of the time, and a negative result 20% of the time. On sites known to be "dry", your test bore procedure has yielded a positive result 10% of the time, an inconclusive result 30% of the time, and a negative result 60% of the time.

(a) Compute the following:

- **The probabilities the test bore will yield positive, inconclusive, or negative result on this plot of land**
- **The probabilities that the well drilled on the site will have gas, given a positive, inconclusive, or negative result for the test bore, respectively.**

(b) Assuming that you wish to be guided by the EMV decision criterion, develop a decision tree that tells you the optimal course of action to take. Describe the optimal policy and its EMV.

(c) What is the EVSI of the test bore procedure?

(d) What is the EVPI in this situation?

Q2:   Product Rollouts

Your firms has developed a new product aimed at the European and Asian markets. For each of these two markets, you have identified two possible sales scenarios, called "good" and "bad", with the following probabilities:

	Europe Good	Europe Bad
Asia Good	0.55	0.15
Asia Bad	0.20	0.10

That is, there is a 55% chance the products sales will be good in Asia and Europe, a 15% chance they will be good in Asia but bad in Europe, and so forth.

You have four possible courses of action:

- Introduce the product simultaneously in Europe and Asia
- Introduce it in Asia first. After it becomes apparent whether sales are good or bad, decide whether to introduce it in Europe, one year later.
- Introduce it in Europe first. After it becomes apparent whether sales are good or bad, decide whether to introduce it in Asia, one year later.
- Abandon the product.

The NPV's of the various scenarios are as follows, in millions of US dollars:

	Immediate Introduction		After one year	
	Good	Bad	Good	Bad
Asia	+120	–205	+117	–205
Europe	+105	–200	+102	–200

For example, "good" sales in Asia mean an NPV of $120 million if the product is introduced in this year, and $117 million in the product is introduced next year. In either year, "bad" sales mean an NPV of –$205 million. The information for Europe should be interpreted similarly.

(a) Calculate:

- **The probability of good sales in Asia**
- **The probability of good sales in Europe**
- **The probability of good sales in Asia, given that good sales are observed in Europe**
- **The probability of good sales in Asia, given that bad sales are observed in Europe**
- **The probability of good sales in Europe, given that good sales are observed in Asia**
- **The probability of good sales in Europe, given that bad sales are observed in Asia.**

(b) Use a decision tree to determine the best introduction strategy for the product from the standpoint of EMV. State the optimal policy and its EMV.

Q3: Airline Yield Management

Bayou Airlines flight 272 flies from a major airport hub to a small community with several industrial facilities. The aircraft used on the flight has 20 seats. Bayou has two fare levels for the flight, a full fare of $259 and a discount fare of $89. Bayou estimates that its incremental cost of flying a full seat versus and empty seat is $11 (this estimates accounts for fuel, complimentary snacks, cleaning etc.). Bayou's strategy is to reserve a certain number of seats q to be sold at full fare, while allowing the remaining seats to be sold at the discount fare. We will assume for simplicity that Bayou's primitive IT systems require that q be determined when seats first go on sale and cannot be changed afterwards. Bayou's experience is that the discount seats sell out essentially immediately after the flight goes on sale. Demand for full-fare seats has been observed to follow the following distribution:

Demand	Probability
1	0.1%
2	0.1%
3	0.7%
4	1.7%
5	3.4%
6	5.8%
7	8.5%
8	10.8%
9	12.2%
10	12.5%

Demand	Probability
11	11.6%
12	9.8%
13	7.7%
14	5.6%
15	3.8%
16	2.4%
17	1.5%
18	0.8%
19	0.4%
20 or more	0.6%

If the full fare seats do not sell out, they fly empty.

From an EMV perspective, what is the optimal number of seats *q* that Bayou should set aside for full fare passengers? Show your work.

Note: the process of maximizing profit by charging different customers different prices for essentially the same product is called *yield management*. The real airline yield management problem is more complicated because there are more than two fare classes and *q* can be revised over time depending on how demand evolves as the time of the flight approaches.

Q4: Supplying Battery Packs

You provide supplies to a weather and ecological research station on a remote island. The number of battery power packs consumed by the station each year has the following distribution:

Packs	Probability
10	11%
11	11%
12	15%
13	15%
14	12%
15	10%
16	8%
17	6%
18	5%
19	5%
20	2%

Once a year, the annual supply boat visits the island, drops of a shipment of battery packs and collects all the battery packs previously on the island, returning them to the mainland for refurbishment. The manufacturer charges you $400 for each battery pack and gives you a credit of $100 for each pack returned to it for refurbishment, regardless of how much it has been used. The boat operator charges you $30 to ship a battery pack to the island, and the same amount to ship one back. If the research station runs out of battery packs between supply boats, you must ship in extra packs by air, and the air taxi operator charges you $150 to ship a pack to the island. All packs shipped in by air return to the mainland on the annual supply boat at the usual $30 cost.

To minimize your expected costs, how many battery packs should you put on the annual supply boat? Show your work.