

Homework Solutions:

Case 6.4, Removing Snow in Montreal

Algebra Formulation:

Question 1 — assigned as homework

Variables:

$x_{ij} = 1$ if we assign sector $i = 1, 2, \dots, 10$ to site $j = 1, 2, \dots, 5$, and 0 otherwise

$$\begin{aligned}
 \min \quad & 153 \cdot 1000 \cdot 0.10(3.4x_{11} + 1.4x_{12} + \dots + 9.3x_{15}) \\
 & + 152 \cdot 1000 \cdot 0.10(2.4x_{12} + 2.1x_{22} + \dots + 8.8x_{25}) \\
 & \quad \vdots \\
 & + 135 \cdot 1000 \cdot 0.10(3.1x_{10,2} + 6.5x_{10,2} + \dots + 8.8x_{10,5}) \\
 \text{ST} \quad & x_{11} + x_{12} + \dots + x_{15} = 1 \\
 & x_{21} + x_{22} + \dots + x_{25} = 1 \\
 & \quad \vdots \\
 & x_{10,1} + x_{10,2} + \dots + x_{10,5} = 1 \\
 & 152x_{11} + 152x_{21} + \dots + 135x_{10,1} \leq 350 \\
 & 152x_{12} + 152x_{22} + \dots + 135x_{10,2} \leq 250 \\
 & \quad \vdots \\
 & 152x_{15} + 152x_{25} + \dots + 135x_{10,5} \leq 200 \\
 & x_{11}, x_{12}, \dots, x_{10,5} \in \{0, 1\}
 \end{aligned}$$

The factor $1000 \cdot 0.01$ (in other words, 100) in the objective function represents that the snow amounts are in 1000's of cubic meters, and it costs \$0.10 to move a cubic meter of snow one kilometer. Thus, this factor converts the whole objective function to (Canadian) dollars. Since this factor is the same for every term in the objective function, it just scales the whole objective function by 100, and may be omitted, leaving the objective function in 1000's of snow-kilometers.

Question 5 — discussed in class

In class, we formulated this model using more advanced mathematical notation (which won't be on the exam). Here's how it would look formulated like the other models we did:

Variables:

$$\begin{aligned}
x_{ij} &= 1 \text{ if we assign sector } i = 1, 2, \dots, 10 \text{ to site } j = 1, 2, \dots, 5, \text{ and } 0 \text{ otherwise} \\
y_j &= 1 \text{ if we expand site } j = 1, 2, \dots, 5, \text{ and otherwise } 0.
\end{aligned}$$

$$\begin{aligned}
\min \quad & 153 \cdot 1000 \cdot 0.10(3.4x_{11} + 1.4x_{12} + \dots + 9.3x_{15}) \\
& + 152 \cdot 1000 \cdot 0.10(2.4x_{12} + 2.1x_{22} + \dots + 8.8x_{25}) \\
& \quad \vdots \\
& + 135 \cdot 1000 \cdot 0.10(3.1x_{10,2} + 6.5x_{10,2} + \dots + 8.8x_{10,5}) \\
\text{ST} \quad & x_{11} + x_{12} + \dots + x_{15} = 1 \\
& x_{21} + x_{22} + \dots + x_{25} = 1 \\
& \quad \vdots \\
& x_{10,1} + x_{10,2} + \dots + x_{10,5} = 1 \\
& 152x_{11} + 152x_{21} + \dots + 135x_{10,1} \leq 350 + 100y_1 \\
& 152x_{12} + 152x_{22} + \dots + 135x_{10,2} \leq 250 + 100y_2 \\
& \quad \vdots \\
& 152x_{15} + 152x_{25} + \dots + 135x_{10,5} \leq 200 + 100y_5 \\
& y_1 + y_2 + \dots + y_5 \leq 1 \\
& x_{11}, x_{12}, \dots, x_{10,5} \in \{0, 1\} \\
& y_1, y_2, \dots, y_5 \in \{0, 1\}
\end{aligned}$$

An alternative approach would be to just solve five modified versions of the original problem, in each case modifying one of the site capacities by 100 units. That would work fine in this case. If the problems were taking a long time to solve, or there were a lot more disposal sites to check, then it would likely be much faster to just add some variables to the integer program, as shown here.

| | A | B | C | D | E | F | G | H | I |
|----|---------------------------------------|----------------------------|----------|----------|----------|----------|-------------------|----------------------------|---|
| 1 | Montreal Snow Removal | | | | | | | | |
| 2 | | | | | | | | | |
| 3 | | Kilometer Distances | | | | | Estimated | | |
| 4 | | Disposal Site | | | | | Snow | | |
| 5 | Sector | 1 | 2 | 3 | 4 | 5 | Amount | (1000 cubic meters) | |
| 6 | 1 | 3.4 | 1.4 | 4.9 | 7.4 | 9.3 | 153 | | |
| 7 | 2 | 2.4 | 2.1 | 8.3 | 9.1 | 8.8 | 152 | | |
| 8 | 3 | 1.4 | 2.9 | 3.7 | 9.4 | 8.6 | 154 | | |
| 9 | 4 | 2.6 | 3.6 | 4.5 | 8.2 | 8.9 | 138 | | |
| 10 | 5 | 1.5 | 3.1 | 2.1 | 7.9 | 8.8 | 127 | | |
| 11 | 6 | 4.2 | 4.9 | 6.5 | 7.7 | 6.1 | 129 | | |
| 12 | 7 | 4.8 | 6.2 | 9.9 | 6.2 | 5.7 | 111 | | |
| 13 | 8 | 5.4 | 6 | 5.2 | 7.6 | 4.9 | 110 | | |
| 14 | 9 | 3.1 | 4.1 | 6.6 | 7.5 | 7.2 | 130 | | |
| 15 | 10 | 3.2 | 6.5 | 7.1 | 6.0 | 8.3 | 135 | | |
| 16 | Site Capacity: | 350 | 250 | 500 | 400 | 200 | | | |
| 17 | | | | | | | | | |
| 18 | Cost per cubic-meter kilometer | | | \$ 0.10 | | | | | |
| 19 | | | | | | | | | |
| 20 | | Assignment Costs | | | | | | | |
| 21 | | Disposal Site | | | | | | | |
| 22 | Sector | 1 | 2 | 3 | 4 | 5 | | | |
| 23 | 1 | 520.2 | 214.2 | 749.7 | 1132.2 | 1422.9 | | | |
| 24 | 2 | 364.8 | 319.2 | 1261.6 | 1383.2 | 1337.6 | | | |
| 25 | 3 | 215.6 | 446.6 | 569.8 | 1447.6 | 1324.4 | | | |
| 26 | 4 | 358.8 | 496.8 | 621.0 | 1131.6 | 1228.2 | | | |
| 27 | 5 | 190.5 | 393.7 | 266.7 | 1003.3 | 1117.6 | | | |
| 28 | 6 | 541.8 | 632.1 | 838.5 | 993.3 | 786.9 | | | |
| 29 | 7 | 532.8 | 688.2 | 1098.9 | 688.2 | 632.7 | | | |
| 30 | 8 | 594.0 | 660.0 | 572.0 | 836.0 | 539.0 | | | |
| 31 | 9 | 403.0 | 533.0 | 858.0 | 975.0 | 936.0 | | | |
| 32 | 10 | 432.0 | 877.5 | 958.5 | 810.0 | 1120.5 | | | |
| 33 | | | | | | | | | |
| 34 | | Site Assignments | | | | | | | |
| 35 | | Disposal Site | | | | | | | |
| 36 | Sector | 1 | 2 | 3 | 4 | 5 | Sum | | |
| 37 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | | |
| 38 | 2 | 1 | 0 | 0 | 0 | 0 | 1 | | |
| 39 | 3 | 0 | 0 | 1 | 0 | 0 | 1 | | |
| 40 | 4 | 0 | 0 | 1 | 0 | 0 | 1 | | |
| 41 | 5 | 0 | 0 | 1 | 0 | 0 | 1 | | |
| 42 | 6 | 0 | 0 | 0 | 1 | 0 | 1 | | |
| 43 | 7 | 0 | 0 | 0 | 1 | 0 | 1 | | |
| 44 | 8 | 0 | 0 | 0 | 0 | 1 | 1 | | |
| 45 | 9 | 1 | 0 | 0 | 0 | 0 | 1 | | |
| 46 | 10 | 0 | 0 | 0 | 1 | 0 | 1 | | |
| 47 | Snow Quantity | 282 | 153 | 419 | 375 | 110 | | | |
| 48 | | | | | | | | | |
| 49 | | | | | | | Total Cost | \$ 547,000 | |

| | A | B | C | D | E | F | G |
|----|---------------------------------------|---------------------------------|------------------------------------|---------------------------------|---------------------------------|---------------------------------|---|
| 1 | Montreal Snow Removal | | | | | | |
| 2 | | | | | | | |
| 3 | | Kilometer Distances | | | | | Estimated |
| 4 | | | | Disposal Site | | | Snow |
| 5 | Sector | 1 | 2 | 3 | 4 | 5 | Amount |
| 6 | 1 | 3.4 | 1.4 | 4.9 | 7.4 | 9.3 | 153 |
| 7 | 2 | 2.4 | 2.1 | 8.3 | 9.1 | 8.8 | 152 |
| 8 | 3 | 1.4 | 2.9 | 3.7 | 9.4 | 8.6 | 154 |
| 9 | 4 | 2.6 | 3.6 | 4.5 | 8.2 | 8.9 | 138 |
| 10 | 5 | 1.5 | 3.1 | 2.1 | 7.9 | 8.8 | 127 |
| 11 | 6 | 4.2 | 4.9 | 6.5 | 7.7 | 6.1 | 129 |
| 12 | 7 | 4.8 | 6.2 | 9.9 | 6.2 | 5.7 | 111 |
| 13 | 8 | 5.4 | 6 | 5.2 | 7.6 | 4.9 | 110 |
| 14 | 9 | 3.1 | 4.1 | 6.6 | 7.5 | 7.2 | 130 |
| 15 | 10 | 3.2 | 6.5 | 7.1 | 6 | 8.3 | 135 |
| 16 | Site Capacity: | =350 | =250 | =500 | =400 | =200 | |
| 17 | | | | | | | |
| 18 | Cost per cubic-meter kilometer | | | 0.1 | | | |
| 19 | | | | | | | |
| 20 | | Assignment Costs | | | | | |
| 21 | | | | Disposal Site | | | |
| 22 | Sector | 1 | 2 | 3 | 4 | 5 | |
| 23 | 1 | =B6*\$G6 | =C6*\$G6 | =D6*\$G6 | =E6*\$G6 | =F6*\$G6 | |
| 24 | 2 | =B7*\$G7 | =C7*\$G7 | =D7*\$G7 | =E7*\$G7 | =F7*\$G7 | |
| 25 | 3 | =B8*\$G8 | =C8*\$G8 | =D8*\$G8 | =E8*\$G8 | =F8*\$G8 | |
| 26 | 4 | =B9*\$G9 | =C9*\$G9 | =D9*\$G9 | =E9*\$G9 | =F9*\$G9 | |
| 27 | 5 | =B10*\$G10 | =C10*\$G10 | =D10*\$G10 | =E10*\$G10 | =F10*\$G10 | |
| 28 | 6 | =B11*\$G11 | =C11*\$G11 | =D11*\$G11 | =E11*\$G11 | =F11*\$G11 | |
| 29 | 7 | =B12*\$G12 | =C12*\$G12 | =D12*\$G12 | =E12*\$G12 | =F12*\$G12 | |
| 30 | 8 | =B13*\$G13 | =C13*\$G13 | =D13*\$G13 | =E13*\$G13 | =F13*\$G13 | |
| 31 | 9 | =B14*\$G14 | =C14*\$G14 | =D14*\$G14 | =E14*\$G14 | =F14*\$G14 | |
| 32 | 10 | =B15*\$G15 | =C15*\$G15 | =D15*\$G15 | =E15*\$G15 | =F15*\$G15 | G37:G46 constrained = 1 |
| 33 | | | | | | | |
| 34 | | Site Assignments | | | | | |
| 35 | | | Changing Cells, constrained binary | | | | |
| 36 | Sector | 1 | 2 | 3 | 4 | 5 | Sum |
| 37 | 1 | 1.22124532709571E-16 | 1 | 0 | 0 | 0 | =SUM(B37:F37) |
| 38 | 2 | 1 | 0 | 0 | 0 | 0 | =SUM(B38:F38) |
| 39 | 3 | 2.0016114846971E-24 | 0 | 1 | 0 | 0 | =SUM(B39:F39) |
| 40 | 4 | 3.09740365307866E-22 | 0 | 1 | 0 | 0 | =SUM(B40:F40) |
| 41 | 5 | 0 | 8.61612005544903E-22 | 1 | 0 | 0 | =SUM(B41:F41) |
| 42 | 6 | 0 | 0 | 0 | 1 | 0 | =SUM(B42:F42) |
| 43 | 7 | 0 | 0 | 0 | 1 | 1.12269088638024E-21 | =SUM(B43:F43) |
| 44 | 8 | 0 | 0 | 9.0770147530029E-23 | 0 | 1 | =SUM(B44:F44) |
| 45 | 9 | 1 | 0 | 0 | 0 | 0 | =SUM(B45:F45) |
| 46 | 10 | 0 | 0 | 0 | 1 | 0 | =SUM(B46:F46) |
| 47 | Snow Quantity | =SUMPRODUCT(B37:B46,\$G6:\$G15) | =SUMPRODUCT(C37:C46,\$G6:\$G15) | =SUMPRODUCT(D37:D46,\$G6:\$G15) | =SUMPRODUCT(E37:E46,\$G6:\$G15) | =SUMPRODUCT(F37:F46,\$G6:\$G15) | |
| 48 | | | | | | | |
| 49 | | | | | | | Total Cost =1000*D18*SUMPRODUCT(B23:F32,B37:F46) |
| 50 | | | | | | | |
| 51 | | | B47:F47 constrained <= B16:F16 | | | | |
| 52 | | | | | Objective, Minimize | | |
| 53 | | | | | | | |

| | A | B | C | D | E | F | G | H | I |
|----|---------------------------------------|----------------------------|----------|----------|----------|-------------------|------------------|----------------------------|---|
| 1 | Montreal Snow Removal | | | | | | | | |
| 2 | | | | | | | | | |
| 3 | | Kilometer Distances | | | | | Estimated | | |
| 4 | | Disposal Site | | | | | Snow | | |
| 5 | Sector | 1 | 2 | 3 | 4 | 5 | Amount | (1000 cubic meters) | |
| 6 | 1 | 3.4 | 1.4 | 4.9 | 7.4 | 9.3 | 153 | | |
| 7 | 2 | 2.4 | 2.1 | 8.3 | 9.1 | 8.8 | 152 | | |
| 8 | 3 | 1.4 | 2.9 | 3.7 | 9.4 | 8.6 | 154 | | |
| 9 | 4 | 2.6 | 3.6 | 4.5 | 8.2 | 8.9 | 138 | | |
| 10 | 5 | 1.5 | 3.1 | 2.1 | 7.9 | 8.8 | 127 | | |
| 11 | 6 | 4.2 | 4.9 | 6.5 | 7.7 | 6.1 | 129 | | |
| 12 | 7 | 4.8 | 6.2 | 9.9 | 6.2 | 5.7 | 111 | | |
| 13 | 8 | 5.4 | 6 | 5.2 | 7.6 | 4.9 | 110 | | |
| 14 | 9 | 3.1 | 4.1 | 6.6 | 7.5 | 7.2 | 130 | | |
| 15 | 10 | 3.2 | 6.5 | 7.1 | 6.0 | 8.3 | 135 | | |
| 16 | Site Capacity: | 350 | 250 | 500 | 400 | 200 | | | |
| 17 | | | | | | | | | |
| 18 | Cost per cubic-meter kilometer | | | \$ 0.10 | | | | | |
| 19 | | | | | | | | | |
| 20 | | Site Assignments | | | | | | | |
| 21 | | Disposal Site | | | | | | | |
| 22 | Sector | 1 | 2 | 3 | 4 | 5 | Sum | | |
| 23 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | | |
| 24 | 2 | 1 | 0 | 0 | 0 | 0 | 1 | | |
| 25 | 3 | 0 | 0 | 1 | 0 | 0 | 1 | | |
| 26 | 4 | 0 | 0 | 1 | 0 | 0 | 1 | | |
| 27 | 5 | 0 | 0 | 1 | 0 | 0 | 1 | | |
| 28 | 6 | 0 | 0 | 0 | 1 | 0 | 1 | | |
| 29 | 7 | 0 | 0 | 0 | 1 | 0 | 1 | | |
| 30 | 8 | 0 | 0 | 0 | 0 | 1 | 1 | | |
| 31 | 9 | 1 | 0 | 0 | 0 | 0 | 1 | | |
| 32 | 10 | 0 | 0 | 0 | 1 | 0 | 1 | | |
| 33 | Snow Quantity | 282 | 153 | 419 | 375 | 110 | Sum | | |
| 34 | 1000 Snow-Kilometers | 767.8 | 214.2 | 1457.5 | 2491.5 | 539.0 | 5470.0 | | |
| 35 | | | | | | | | | |
| 36 | | | | | | Total Cost | \$ 547,000 | | |

| | A | B | C | D | E | F | G |
|----|---------------------------|--|--|--|--|--|---------------|
| 1 | Montreal Snow Removal | | | | | | |
| 2 | | | | | | | |
| 3 | | Kilometer Distances | | | | | Estimated |
| 4 | | | | Disposal Site | | | Snow |
| 5 | Sector | 1 | 2 | 3 | 4 | 5 | Amount |
| 6 | 1 | 3.4 | 1.4 | 4.9 | 7.4 | 9.3 | 153 |
| 7 | 2 | 2.4 | 2.1 | 8.3 | 9.1 | 8.8 | 152 |
| 8 | 3 | 1.4 | 2.9 | 3.7 | 9.4 | 8.6 | 154 |
| 9 | 4 | 2.6 | 3.6 | 4.5 | 8.2 | 8.9 | 138 |
| 10 | 5 | 1.5 | 3.1 | 2.1 | 7.9 | 8.8 | 127 |
| 11 | 6 | 4.2 | 4.9 | 6.5 | 7.7 | 6.1 | 129 |
| 12 | 7 | 4.8 | 6.2 | 9.9 | 6.2 | 5.7 | 111 |
| 13 | 8 | 5.4 | 6 | 5.2 | 7.6 | 4.9 | 110 |
| 14 | 9 | 3.1 | 4.1 | 6.6 | 7.5 | 7.2 | 130 |
| 15 | 10 | 3.2 | 6.5 | 7.1 | 6 | 8.3 | 135 |
| 16 | Site Capacity: | =350 | =250 | =500 | =400 | =200 | |
| 17 | | | | | | | |
| 18 | Cost per cubic-meter kilc | | | 0.1 | | | |
| 19 | | | Changing Cells, constrained binary | | | G23:G32 constrained = 1 | |
| 20 | | Site Assignments | | | | | |
| 21 | | | | Disposal Site | | | |
| 22 | Sector | 1 | 2 | 3 | 4 | 5 | Sum |
| 23 | 1 | 1.11022302470556E-17 | 1 | 0 | 0 | 0 | =SUM(B23:F23) |
| 24 | 2 | 1 | 0 | 0 | 0 | 0 | =SUM(B24:F24) |
| 25 | 3 | 2.0016114846971E-24 | 0 | 1 | 0 | 0 | =SUM(B25:F25) |
| 26 | 4 | 3.09740365307866E-22 | 0 | 1 | 0 | 0 | =SUM(B26:F26) |
| 27 | 5 | 0 | 8.61612005544903E-22 | 1 | 0 | 0 | =SUM(B27:F27) |
| 28 | 6 | 0 | 0 | 0 | 1 | 0 | =SUM(B28:F28) |
| 29 | 7 | 0 | 0 | 0 | 1 | 1.12269088638024E-21 | =SUM(B29:F29) |
| 30 | 8 | 0 | 0 | 9.0770147530029E-23 | 0 | 1 | =SUM(B30:F30) |
| 31 | 9 | 1 | 0 | 0 | 0 | 0 | =SUM(B31:F31) |
| 32 | 10 | 0 | 0 | 0 | 1 | 0 | =SUM(B32:F32) |
| 33 | Snow Quantity | =SUMPRODUCT(B23:B32,\$G6:\$G15) | =SUMPRODUCT(C23:C32,\$G6:\$G15) | =SUMPRODUCT(D23:D32,\$G6:\$G15) | =SUMPRODUCT(E23:E32,\$G6:\$G15) | =SUMPRODUCT(F23:F32,\$G6:\$G15) | Sum |
| 34 | 1000 Snow-Kilometers | =SUMPRODUCT(B23:B32,B6:B15,\$G6:\$G15) | =SUMPRODUCT(C23:C32,C6:C15,\$G6:\$G15) | =SUMPRODUCT(D23:D32,D6:D15,\$G6:\$G15) | =SUMPRODUCT(E23:E32,E6:E15,\$G6:\$G15) | =SUMPRODUCT(F23:F32,F6:F15,\$G6:\$G15) | =SUM(B34:F34) |
| 35 | | | | | | | |
| 36 | | Note: C33:F34 contain copies of B33:B34 (I made the columns narrower for printing clarity) | | | | | Total Cost |
| 37 | | | | B33:F33 constrained <= B16:F16 | | | =D18*1000*G34 |
| 38 | | | | | | Objective, Minimize | |
| 39 | | | | | | | |
| 40 | | | | | | | |
| 41 | | | | | | | |

| | A | B | C | D | E | F | G | H | I |
|----|--------------------------------|---------------------|-------|---------|--------|-------|------------|---------------------|---|
| 1 | Montreal Snow Removal | | | | | | | | |
| 2 | | | | | | | | | |
| 3 | | Kilometer Distances | | | | | Estimated | | |
| 4 | | Disposal Site | | | | | Snow | | |
| 5 | Sector | 1 | 2 | 3 | 4 | 5 | Amount | (1000 cubic meters) | |
| 6 | 1 | 3.4 | 1.4 | 4.9 | 7.4 | 9.3 | 153 | | |
| 7 | 2 | 2.4 | 2.1 | 8.3 | 9.1 | 8.8 | 152 | | |
| 8 | 3 | 1.4 | 2.9 | 3.7 | 9.4 | 8.6 | 154 | | |
| 9 | 4 | 2.6 | 3.6 | 4.5 | 8.2 | 8.9 | 138 | | |
| 10 | 5 | 1.5 | 3.1 | 2.1 | 7.9 | 8.8 | 127 | | |
| 11 | 6 | 4.2 | 4.9 | 6.5 | 7.7 | 6.1 | 129 | | |
| 12 | 7 | 4.8 | 6.2 | 9.9 | 6.2 | 5.7 | 111 | | |
| 13 | 8 | 5.4 | 6 | 5.2 | 7.6 | 4.9 | 110 | | |
| 14 | 9 | 3.1 | 4.1 | 6.6 | 7.5 | 7.2 | 130 | | |
| 15 | 10 | 3.2 | 6.5 | 7.1 | 6.0 | 8.3 | 135 | | |
| 16 | Site Capacity: | 350 | 250 | 500 | 400 | 200 | | | |
| 17 | | | | | | | | | |
| 18 | Cost per cubic-meter kilometer | | | \$ 0.10 | | | | | |
| 19 | | | | | | | | | |
| 20 | Site expansion amount | | | 100 | | | | | |
| 21 | Number of sites to expand | | | 1 | | | | | |
| 22 | | | | | | | | | |
| 23 | | Site Assignments | | | | | | | |
| 24 | | Disposal Site | | | | | | | |
| 25 | Sector | 1 | 2 | 3 | 4 | 5 | Sum | | |
| 26 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | | |
| 27 | 2 | 1 | 0 | 0 | 0 | 0 | 1 | | |
| 28 | 3 | 1 | 0 | 0 | 0 | 0 | 1 | | |
| 29 | 4 | 0 | 0 | 1 | 0 | 0 | 1 | | |
| 30 | 5 | 0 | 0 | 1 | 0 | 0 | 1 | | |
| 31 | 6 | 0 | 0 | 0 | 0 | 1 | 1 | | |
| 32 | 7 | 0 | 0 | 0 | 1 | 0 | 1 | | |
| 33 | 8 | 0 | 0 | 1 | 0 | 0 | 1 | | |
| 34 | 9 | 1 | 0 | 0 | 0 | 0 | 1 | | |
| 35 | 10 | 0 | 0 | 0 | 1 | 0 | 1 | | |
| 36 | Snow Quantity | 436 | 153 | 375 | 246 | 129 | Sum | | |
| 37 | 1000 Snow-Kilometers | 983.4 | 214.2 | 1459.7 | 1498.2 | 786.9 | 4942.4 | | |
| 38 | | | | | | | | | |
| 39 | | 1 | 2 | 3 | 4 | 5 | Sum | | |
| 40 | Expand site? | 1 | 0 | 0 | 0 | 0 | 1 | | |
| 41 | Expanded size | 450 | 250 | 500 | 400 | 200 | | | |
| 42 | | | | | | | | | |
| 43 | | | | | | | Total Cost | \$ 494,240 | |

| | A | B | C | D | E | F | G |
|----|--------------------------------|--|-----------------|--------------------------------|-----------------|-------------------------|---------------|
| 1 | Montreal Snow Removal | | | | | | |
| 2 | | | | | | | |
| 3 | | Kilometer Distances | | | | | Estimated |
| 4 | | | | Disposal Site | | | Snow |
| 5 | Sector | 1 | 2 | 3 | 4 | 5 | Amount |
| 6 | 1 | 3.4 | 1.4 | 4.9 | 7.4 | 9.3 | 153 |
| 7 | 2 | 2.4 | 2.1 | 8.3 | 9.1 | 8.8 | 152 |
| 8 | 3 | 1.4 | 2.9 | 3.7 | 9.4 | 8.6 | 154 |
| 9 | 4 | 2.6 | 3.6 | 4.5 | 8.2 | 8.9 | 138 |
| 10 | 5 | 1.5 | 3.1 | 2.1 | 7.9 | 8.8 | 127 |
| 11 | 6 | 4.2 | 4.9 | 6.5 | 7.7 | 6.1 | 129 |
| 12 | 7 | 4.8 | 6.2 | 9.9 | 6.2 | 5.7 | 111 |
| 13 | 8 | 5.4 | 6 | 5.2 | 7.6 | 4.9 | 110 |
| 14 | 9 | 3.1 | 4.1 | 6.6 | 7.5 | 7.2 | 130 |
| 15 | 10 | 3.2 | 6.5 | 7.1 | 6 | 8.3 | 135 |
| 16 | Site Capacity: | =350 | =250 | =500 | =400 | =200 | |
| 17 | | | | | | | |
| 18 | Cost per cubic-meter kilometer | | | 0.1 | | | |
| 19 | | | | | | | |
| 20 | Site expansion amount | | | 100 | | | |
| 21 | Number of sites to expand | Changing Cells, constrained binary | | 1 | | G23:G32 constrained = 1 | |
| 22 | | | | | | | |
| 23 | | Site Assignments | | | | | |
| 24 | | | | Disposal Site | | | |
| 25 | Sector | 1 | 2 | 3 | 4 | 5 | Sum |
| 26 | 1 | 1.2490009027033E-16 | 1 | 0 | 0 | 0 | =SUM(B26:F26) |
| 27 | 2 | 1 | 0 | 0 | 0 | 0 | =SUM(B27:F27) |
| 28 | 3 | 0.999999999993339 | 0 | 0 | 0 | 0 | =SUM(B28:F28) |
| 29 | 4 | 3.09740365307866E-22 | 0 | 1 | 0 | 0 | =SUM(B29:F29) |
| 30 | 5 | 8.61612005544903E-22 | 0 | 1 | 0 | 0 | =SUM(B30:F30) |
| 31 | 6 | -6.6613381477073E-12 | 0 | 0 | 0 | 1 | =SUM(B31:F31) |
| 32 | 7 | 1.12269088638024E-21 | 0 | 0 | 1 | 0 | =SUM(B32:F32) |
| 33 | 8 | 0 | 0 | 0.999999999993339 | 0 | 0 | =SUM(B33:F33) |
| 34 | 9 | 1 | 0 | 0 | 0 | 0 | =SUM(B34:F34) |
| 35 | 10 | 0 | 0 | 0 | 1 | 0 | =SUM(B35:F35) |
| 36 | Snow Quantity | =SUMPRODUCT(B26:B35,\$G6:\$G15) | =SUMPRODUCT(C26 | =SUMPRODUCT(D26 | =SUMPRODUCT(E26 | =SUMPRODUCT(F26 | Sum |
| 37 | 1000 Snow-Kilometers | =SUMPRODUCT(B26:B35,B6:B15,\$G6:\$G15) | =SUMPRODUCT(C26 | =SUMPRODUCT(D26 | =SUMPRODUCT(E26 | =SUMPRODUCT(F26 | =SUM(B37:F37) |
| 38 | | Changing Cells, constrained binary | | | | | |
| 39 | | 1 | 2 | 3 | 4 | 5 | Sum |
| 40 | Expand site? | 1 | 0 | 0 | 0 | 0 | =SUM(B40:F40) |
| 41 | Expanded size | =B16+D\$20*B40 | =C16+E\$20*C40 | =D16+F\$20*D40 | =E16+G\$20*E40 | =F16+H\$20*F40 | |
| 42 | | | | | | | |
| 43 | | | | | | | Total Cost |
| 44 | | | | | | | =D18*1000*G37 |
| 45 | | | | | | | |
| 46 | | Note: C36:F37 contain copies of B36:B37 (I made the columns narrower for printing clarity) | | B36:F36 constrained <= B41:F41 | | Objective, Minimize | |
| 47 | | | | | | | |
| 48 | | | | | | | |

```

### -----
### Parameters of model.  Actual values come from .dat file
### -----

set SECTOR ordered;
set SITE    ordered;

param snowAmount {SECTOR} >= 0;
param capacity {SITE} >= 0;

param distance {SECTOR, SITE} >= 0;

### -----
### Decision variables
### -----

var x {SECTOR, SITE} binary;

### -----
### Objective function
### -----

minimize totalCost:
    sum {i in SECTOR, j in SITE}
        1000 * 0.1 * snowAmount[i] * distance[i,j] * x[i,j];

### -----
### Constraints (binary constraints already defined with variables)
### -----

subject to oneSite {i in SECTOR}:
    sum {j in SITE} x[i,j] = 1;

subject to siteCapacity {j in SITE}:
    sum {i in SECTOR} snowAmount[i]*x[i,j] <= capacity[j];

```

data;

param: SECTOR: snowAmount := # defines set "SECTOR" and param "snowAmount"
Sec1 153 Sec2 152 Sec3 154 Sec4 138 Sec5 127
Sec6 129 Sec7 111 Sec8 110 Sec9 130 Sec10 135 ;

param: SITE: capacity := # defines "SITE" and "capacity"
Site1 350 Site2 250 Site3 500 Site4 400 Site5 200 ;

param distance:
Site1 Site2 Site3 Site4 Site5 :=
Sec1 3.4 1.4 4.9 7.4 9.3
Sec2 2.4 2.1 8.3 9.1 8.8
Sec3 1.4 2.9 3.7 9.4 8.6
Sec4 2.6 3.6 4.5 8.2 8.9
Sec5 1.5 3.1 2.1 7.9 8.8
Sec6 4.2 4.9 6.5 7.7 6.1
Sec7 4.8 6.2 9.9 6.2 5.7
Sec8 5.4 6.0 5.2 7.6 4.9
Sec9 3.1 4.1 6.6 7.5 7.2
Sec10 3.2 6.5 7.1 6.0 8.3 ;

```

### -----
### Parameters of model.  Actual values come from .dat file
### -----

set SECTOR ordered;
set SITE    ordered;

param snowAmount {SECTOR} >= 0;
param capacity {SITE} >= 0;

param distance {SECTOR, SITE} >= 0;

param expansionSize >= 0;
param numToExpand integer, >= 0;

### -----
### Decision variables
### -----

var x {SECTOR, SITE} binary;
var y {SITE} binary;

### -----
### Objective function
### -----

minimize totalCost:
    sum {i in SECTOR, j in SITE}
        1000 * 0.1 * snowAmount[i] * distance[i,j] * x[i,j];

### -----
### Constraints (binary constraints already defined with variables)
### -----

subject to oneSite {i in SECTOR}:
    sum {j in SITE} x[i,j] = 1;

subject to siteCapacity {j in SITE}:
    sum {i in SECTOR} snowAmount[i]*x[i,j]
        <= capacity[j] + expansionSize*y[j];

subject to numExpansions:
    sum {j in SITE} y[j] <= numToExpand;

```

data;

param: SECTOR: snowAmount := # defines set "SECTOR" and param "snowAmount"
Sec1 153 Sec2 152 Sec3 154 Sec4 138 Sec5 127
Sec6 129 Sec7 111 Sec8 110 Sec9 130 Sec10 135 ;

param: SITE: capacity := # defines "SITE" and "capacity"
Site1 350 Site2 250 Site3 500 Site4 400 Site5 200 ;

param distance:
Site1 Site2 Site3 Site4 Site5 :=
Sec1 3.4 1.4 4.9 7.4 9.3
Sec2 2.4 2.1 8.3 9.1 8.8
Sec3 1.4 2.9 3.7 9.4 8.6
Sec4 2.6 3.6 4.5 8.2 8.9
Sec5 1.5 3.1 2.1 7.9 8.8
Sec6 4.2 4.9 6.5 7.7 6.1
Sec7 4.8 6.2 9.9 6.2 5.7
Sec8 5.4 6.0 5.2 7.6 4.9
Sec9 3.1 4.1 6.6 7.5 7.2
Sec10 3.2 6.5 7.1 6.0 8.3 ;

param expansionSize := 100 ;
param numToExpand := 1 ;