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Review Material for First Exam

The exam is scheduled to take 90 minutes. These questions are taken from old exams in a related course. I apologize that the may not be as representative as I would like of the questions I might give on the exam, since I am covering a different mix of material in this class. In particular, the exam might include nonlinear problems, which are not represented here.

Here are some grading points to be aware of:

- In algebra models, define all your decision variables precisely as measurable numbers like "the number of gallons of yogurt produced," not just "yogurt".
- In the spreadsheet problems, try to refer to the *given data* of the model using cell references. For example, if the unit cost of labor is \$25/hour, shown in the spreadsheet as a 25 in cell C3, and the hours of labor used is in E20, then C3*E20 would be a better formula for labor cost than 25*E20. The reason is that if the cost of labor changes, somebody using the spreadsheet can just change the obvious data in C3 and not have to examine every formula in the spreadsheet.
- Similarly, you should not "bury" given data in the problem inside Solver constraints. For example if the total number of labor hours available is 160, and this value is in cell F20, the constraint G18 <= F20 is better practice than G18 <= 160.
- Sometimes I will ask that you write formulas that will yield correct results when copied to other cells. That means you have to correctly decide whether to put \$ absolute references into your formulas, and where.
- There will be small point deductions for not using correct syntax when writing Excel formulas.

1: Snowboard Luggage

Breakneck Corporation makes specialized cloth luggage for snowboarders. They have three products, *small*, *medium* and *large*, which bring revenues of \$25, \$40, and \$50 per unit, respectively. Production of each piece of luggage consumes two resources, *labor* and *fabric*. The following table gives information about the products:

	Small	**Medium**	**Large**
Labor Consumed (hours)	0.8	1	1.1
Fabric Consumed (yards)	3	5	8
Product Weight (lbs)	4.4	5.1	5.9

In the coming month, up to 2,500 hours of labor are available at \$12.00 an hour. The firm has a discount contract with its fabric supplier, allowing it to purchase up to 9,000 yards of fabric per month at \$3.00 per yard.

The firm's policy is to exactly meet its distributors' demand for each of the three kinds of luggage. If the firm spends nothing on promotions, next month's demand for the small, medium, and large products will be 500, 1000, and 750 units, respectively. The firm can cause an increase in these demands by spending money on either (or both) of two promotion programs: *trade promotions* and *direct customer promotions*. Each dollar spent on trade promotions causes an expected increase in demand of 0.10 unit for each type of luggage: for example, spending $20 on trade promotions would stimulate additional demand for $20 \times 0.10 = 2$ small units, 2 medium units, and 2 large units. Each dollar spent on direct customer promotions causes an expected sales increase of 0.12 small units, 0.10 medium units, and 0.08 large units.

Algebraically formulate a linear programming model to maximize the firm's profits for the coming month. Give clear, numeric definitions of your decision variables. You are allowed to skip algebraic simplifications (if any arise).

2: Production, Inventory and Advertising

You are in charge of production planning for a firm that make computer game joysticks, which you sell for $34.50 each. Over the next six months, you expect demand and unit production costs to vary as follows:

Month	Base Demand	Production Unit Cost
1	3,000	$ 25.00
2	4,000	$ 26.00
3	6,000	$ 28.50
4	2,000	$ 29.00
5	1,500	$ 32.50
6	2,500	$ 26.50

Demand is stimulated above the "base" levels shown in this table by spending money on advertising. Each month, you can spend up to $900 on advertising. Each dollar of advertising stimulates an additional demand of two units in the month it is spent, and one unit in the following month. Last month, in fact, you spent $400 on advertising, so actual demand for month one will be at least 3,400 units (and possibly more, if you spend any money on advertising in month one). You are committed to meeting all the demand for your product.

Your production line has a regular capacity of 4,500 units per month. An extra 500 units per month can be produced in addition to this amount, but the unit cost is $5.00 higher than shown above.

Up to 1,000 units can be kept in inventory. Inventory cost is $1.50 per unit per month, assessed on the amount of inventory remaining at the end of each month. Your starting inventory is 300 units.

You are using the spreadsheet on the next page to try to plan production for the next six months. The shaded cells contain formulas. Cell E14 contains the formula =D5, and cell I14 contains the formula =D10. The changing cells are E15:G20, and the optimal solution is shown.

(a) What formula should be in cell H15, to compute the total demand for joysticks in month one? Make sure your answer will yield correct results for the following months when copied to H16:H20 (if you cannot figure out how to do this, give formulas for H16:H20).

(b) What formula should be in cell I15, to compute the inventory remaining at the end of month one? Make sure your answer will yield correct results for the following months when copied to I16:I20 (if you cannot figure out how to do this, give formulas for I16:I20).

(c) What formulas should be in cells D22 (revenue), D23 (total cost for regular production), D24 (total cost for extra production), D25 (total advertising cost), D26 (total inventory cost), and D27 (total profit)?

(d) What is the target cell? Should it be maximized or minimized? What are all the constraints that should be used in the solver? Should you "assume linear model"? Should you "assume nonnegative"?

	A	B	C	D	E	F	G	H	I
1			**Selling Price**	$ 34.50					
2			**Inventory Unit Cost**	$ 1.50					
3			**Increased Demand per $ Advertising**	2	**(current month)**				
4			**Increased Demand per $ Advertising**	1	**(following month)**				
5			**Previous Month's Advertising**	$ 400.00					
6			**Max Advertising**	$ 900.00					
7			**Production Capacity (Regular)**	4,500					
8			**Production Capacity (Extra)**	500					
9			**Additional Unit Cost for Extra**	$ 5.00					
10			**Current Inventory**	300					
11			**Max Inventory**	1000					
12						**Regular**	**Extra**		**End**
13	**Month**	**Base Demand**	**Unit Cost**		**Advertising**	**Production**	**Production**	**Demand**	**Inventory**
14	Prior				$ 400.00				300
15	1	3,000	$ 25.00		$ 633.33	4,500	500	4,667	633
16	2	4,000	$ 26.00		$ -	4,500	500	4,633	1,000
17	3	6,000	$ 28.50		$ -	4,500	500	6,000	0
18	4	2,000	$ 29.00		$ 900.00	4,500	0	3,800	700
19	5	1,500	$ 32.50		$ 900.00	3,700	0	4,200	200
20	6	2,500	$ 26.50		$ 900.00	4,500	500	5,200	0
21									
22			**Revenue**	$ 983,250					
23			**Production Cost (Regular)**	$ 727,750					
24			**Production Cost (Extra)**	$ 63,000					
25			**Advertising Cost**	$ 3,333					
26			**Inventory Cost**	$ 3,800					
27			**Profit**	$ 185,367					

3: Production, Shipping, and Marketing (Algebra)

Your firm designs and markets specialty personal electronic devices. You can use any of three different overseas suppliers to manufacture the devices. After manufacture, you ship the products to three different regional distributors, who pay you $49.95 per device. The unit manufacturing costs for the three suppliers and their unit shipping costs to each of the three distributors are shown in the following table:

Supplier	Unit Shipping Cost to Distributor 1	2	3	Unit Manufacturing Cost	Availability
1	$ 2.30	$ 3.25	$ 3.50	$ 23.50	25,000
2	$ 4.15	$ 2.00	$ 3.85	$ 17.90	10,000
3	$ 3.95	$ 3.20	$ 2.85	$ 21.00	12,000

The "availability" column gives the maximum number of units that can be ordered from each supplier.

The amount shipped to each distributor should be the same as that distributor's demand, which depends on your advertising expenditures. If you spend nothing on advertising, the three distributors' demand levels would be 8,000 units, 5,000 units, and 7,500 units, respectively.

Each $100 you spend on *national* advertising increases distributor 1's demand by 2 units, distributor 2's demand by 3 units, and distributor 3's demand by 4 units.

You can also spend money on *regional* advertising, which is separate for each distributor's region: each $100 spent in distributor 1's region increases distributor 1's demand by 8 units, and has no significant effect on the other distributors' demands. Similarly, every $100 spent on advertising in distributor 2's region raises demand there by 6 units, and every $100 spent in distributor 3's region raises distributor 3's demand by 5 units.

Algebraically formulate a linear programming model to give your firm the highest possible profits. Give clear, numeric definitions of your decision variables. You are allowed to skip algebraic simplifications (if any arise).

4: Making Goat Cheese (Spreadsheet)

Your company, Ma-ma Dairies, makes fresh goat cheese to supply the wholesale restaurant market. You buy goat's milk from a supplier at a price that undergoes significant seasonal variation, as shown in the following table. You process this milk into cheese at a cost of $4.85 per gallon. The milk is very perishable, and you always start processing your entire month's milk purchase as soon as you buy it; you cannot hold milk in inventory.

	Milk Cost per Gallon	Yield (Pounds of Cheese per Gallon of Milk)	Minimum Pounds of Cheese you have to Sell	Maximum Pounds of Cheese you can Sell
Jan	$ 1.90	2.1	700	1,200
Feb	$ 1.85	2.1	1,000	1,500
Mar	$ 1.75	2.3	1,100	2,000
Apr	$ 1.65	2.4	1,200	2,000
May	$ 0.95	2.5	1,100	1,500
Jun	$ 0.85	2.2	700	1,200
Jul	$ 0.85	2.2	800	1,700
Aug	$ 0.85	2.2	900	1,600
Sep	$ 0.90	2.4	1,900	2,400
Oct	$ 1.35	2.5	2,000	2,700
Nov	$ 1.45	2.2	2,000	2,800
Dec	$ 1.80	2.1	2,400	3,000

The *yield* of the processing step – how many pounds of cheese you obtain from a gallon of milk – also varies somewhat on a seasonal basis, as also shown in the table. You can process at most 1,200 gallons of milk per month.

The cheese sells for $4.15 per pound. At this price, you have made firm commitments to sell your customers various amounts of cheese, varying by month, as shown in the "minimum" column in the table. Your customers will also buy more cheese if you produce it, up to the amount shown in the "maximum" column:

Unlike milk, cheese can be held in inventory in a special "controlled atmosphere" refrigerator. The cost is $0.85 per pound per month, assessed on the amount of inventory held at the end of each month. You expect to have 80 pounds of cheese in inventory at the beginning of the year.

You are using the spreadsheet on the next page of this material to help plan the coming year to obtain the maximum possible profit (you may detach the last page). Cell B25 contains the formula =D21, and cell D25 contains =D22.

(a) What formula should be in cell D24, to compute the amount of cheese left in inventory at the end of January? Make sure your answer will yield correct results for rest of the year when copied to cells D25:D35.

(b) What formula should be in cell B36, to compute the total amount of milk ordered for the year?

(c) What formula should be in cell C36, to compute the total cheese sold for the year?

(d) What formulas should be in cells D38:D42, to compute the revenue, costs, and profit for your plan?

(e) What is the target cell? Should it be maximized or minimized? What are the changing cells? What are all the constraints that should be used in the solver? Should you "assume linear model"? Should you "assume nonnegative"?

	A	B	C	D	E	F
1		**Milk Cost per Gallon**	**Yield (Pounds Cheese per Gallon of Milk)**	**Minimum Pounds Cheese to Sell**	**Maximum Cheese to Sell**	
2	**Jan**	$ 1.90	2.1	700	1,200	
3	**Feb**	$ 1.85	2.1	1,000	1,500	
4	**Mar**	$ 1.75	2.3	1,100	2,000	
5	**Apr**	$ 1.65	2.4	1,200	2,000	
6	**May**	$ 0.95	2.5	1,100	1,500	
7	**Jun**	$ 0.85	2.2	700	1,200	
8	**Jul**	$ 0.85	2.2	800	1,700	
9	**Aug**	$ 0.85	2.2	900	1,600	
10	**Sep**	$ 0.90	2.4	1,900	2,400	
11	**Oct**	$ 1.35	2.5	2,000	2,700	
12	**Nov**	$ 1.45	2.2	2,000	2,800	
13	**Dec**	$ 1.80	2.1	2,400	3,000	
14						
15			**Cheese Price**	$ 4.15	per pound	
16			**Processing Cost**	$ 4.85	per gallon	
17			**Max Milk Processed**	1,200.0	gallons per month	
18			**Inventory Cost**	$ 0.10	per pound per month	
19		**Prior December's Milk Purchase**		400.0	gallons	
20		**Starting Cheese Inventory**		80.0	pounds	
21						
22		**Gallons Milk Bought**	**Pounds Cheese Sold**	**Pounds Cheese In Inventory**		
23	**Prior Dec**			80.0		
24	**Jan**	533.3	1,200.0	0.0		
25	**Feb**	714.3	1,500.0	0.0		
26	**Mar**	869.6	2,000.0	0.0		
27	**Apr**	833.3	2,000.0	0.0		
28	**May**	1,200.0	1,500.0	1,500.0		
29	**Jun**	0.0	1,200.0	300.0		
30	**Jul**	1,200.0	1,700.0	1,240.0		
31	**Aug**	1,200.0	1,600.0	2,280.0		
32	**Sep**	1,200.0	2,400.0	2,760.0		
33	**Oct**	1,200.0	2,700.0	3,060.0		
34	**Nov**	1,200.0	2,800.0	2,900.0		
35	**Dec**	47.6	3,000.0	-0.0		
36	**Total**	10,198.1	23,600.0			
37						
38			**Revenue**	$ 97,940.00		
39			**Milk Cost**	$ 12,937.22		
40			**Processing Cost**	$ 49,460.96		
41			**Inventory Cost**	$ 1,404.00		
42			**Profit**	$ 34,137.82		

5: Choosing a Class Schedule

Shelly Shaw is a marketing senior in the business program at Relatively Normal University (RNU). With the help of her roommate Anny Litical, Shelly is trying to decide what courses to take next semester. Shelly has identified ten possible courses she might want to take and has assigned a "point score" between 1 and 10 to each one, with 10 indicating the most desirable course, and 1 the least. The "homework" column is an estimate of the average hours of homework assigned per week.

	Course	Instructor	Credits	Homework	Points	Time
1	Intermediate Marketing (section 1)	Neisgye	3	6	10	MW 10:30AM
2	Intermediate Marketing (section 2)	Meeney	3	8	8	MW 1:30PM
3	Intermediate Marketing (section 3)	Yewslis	3	5	4	MW 3:00PM
4	International Marketing	Neisgye	4	5	9	MW 3:00PM
5	Market Research	Keiskware	3	8	7	TTh 10:30AM
6	Internet Marketing Project	Jones	4	7	9	MW 10:30AM
7	Sales Management	Bosman	3	4	8	TTh 1:30PM
8	Advanced MIS for Non-Majors	Phillips	3	5	7	TTh 10:30AM
9	Cases in International Management	Hunter	3	4	6	TTh 3:00PM
10	Chinese I	Chen	4	8	6	MW 1:30PM

Shelly has to take at least 14 and at most 17 credits, and wants an average homework load no more than 30 hours per week. She has to take *Intermediate Marketing*, but can choose any section. She must also take at least one Marketing elective, and at least one general elective. The marketing electives are *International Marketing*, *Market Research*, *Internet Marketing Project*, and *Sales Management*. The general electives are *Advanced MIS for Non-Majors*, *Cases in International Management*, and *Chinese I*. The university will not let Shelly register for two courses that meet at the same time.

Write an algebraic integer programming model for choosing a schedule that has the highest possible total point score, subject to the constraints described above. Clearly define your variables. The total point score of a schedule is the sum of the point scores of the classes being taken.

6: Hiring Consultants

Your firm is forming a panel of three to five consultants to advise it on its latest product development project. Each candidate to be on the panel has been classified as being competent in one or more areas of expertise: *computer systems* (CS), *management* (MGMT), *marketing* (MKT), and *operations analysis* (OA). There must be at least one panelist competent in each area of expertise, except for marketing, for which there must be at least two. The following table describes the available candidates:

	Hourly Rate	Expertise
Joe Nowital	$ 250	CS, MGMT, MKT
John Ecklestone	$ 150	CS, OA
Mary Hacker	$ 125	CS
Phil Saftee	$ 185	MGMT, MKT
Max Bradley	$ 200	MGMT, OA
Sarah Lyddle	$ 190	MGMT, MKT, OA

Due to an old academic squabble, Max Bradley and Sarah Lyddle dislike one another. If one of them is hired, the other cannot be.

Algebraically formulate an integer program that will find a panel that meets all the constraints above and has the lowest cost, where cost is defined to be the sum of the hourly rates of the consultants hired. Clearly define your variables.

7: Planning an Advertising Campaign

Your firm makes a product that costs $15.50 per unit to produce, and sells for $23.00. You have up to $30,000 to spend on an advertising campaign for the product, and you have identified six possible magazines in which to place your advertisements: *Motor Sport*, *Off-Road*, *Trout Caster*, *DotComWorld*, *Mad Hacker*, and *Downhill Ski*. You can run up to five ads in each magazine, with costs as follows:

	Magazine					
	Motor Sport	**Off-Road**	**Trout Caster**	**DotComWorld**	**Mad Hacker**	**Downhill Ski**
Cost per Ad	$ 2,000	$ 1,600	$ 2,000	$ 1,400	$ 1,000	$ 1,500

Your marketing department has estimated the number of sales that would result from placing one, two, three, four, or five ads in each magazine, as follows:

	Sales Generated					
Number of Ads	**Motor Sport**	**Off-Road**	**Trout Caster**	**DotComWorld**	**Mad Hacker**	**Downhill Ski**
1	4,000	2,000	4,500	2,300	2,000	3,500
2	6,000	3,000	5,500	3,100	2,500	5,000
3	7,200	3,500	6,000	3,900	3,000	5,200
4	8,000	3,700	6,200	4,700	3,100	5,400
5	8,200	3,900	6,250	5,000	3,200	5,600

For instance, placing one ad in *Motor Sport* would yield 4,000 sales of the product, while placing two ads in *Motor Sport* would produce a further 2,000 sales, for a total of 6,000. A third ad would generate another 1,200 sales, for a total of 7,200, and so forth.

You would like to plan the advertising campaign so that you generate the greatest possible net profit for the company, given that your ad expenditures stay within the budget. To do so, you are using the spreadsheet shown on the next page. The shaded cells contain formulas. The only changing cells are B20:G24, and are interpreted as follows: in the column corresponding to a given magazine, a one appears in the row corresponding to the number of ads to be placed, and the rest of the changing cells in the column contain zeroes. If a magazine is not used, all the changing cells in its column should be zero.

(a) Cell B25 should contain a one if any ads are placed in *Motor Sport*, and otherwise it should contain a zero. It is not itself a changing cell, but is computed via a formula. What should that formula be? Make sure your answer will yield correct results for the other magazines when copied to cells C25:G25.

(b) What formula should you put in cell B28, the total amount spent to place ads in *Motor Sport*? Make sure your answer will yield correct results for the other magazines when copied to cells C28:G28.

	A	B	C	D	E	F	G
1		Magazine					
2		Motor Sport	Off-Road	Field & Stream	Dot-Com World	Mad Hacker	Downhill Ski
3	Cost per Ad	$ 2,000	$ 1,600	$ 2,000	$ 1,400	$ 1,000	$ 1,500
4							
5	Added Sales (Units):						
6							
7	Number of Ads	Motor Sport	Off-Road	Field & Stream	Dot-Com World	Mad Hacker	Downhill Ski
8	1	4,000	2,000	4,500	2,300	2,000	3,500
9	2	6,000	3,000	5,500	3,100	2,500	5,000
10	3	7,200	3,500	6,000	3,900	3,000	5,200
11	4	8,000	3,700	6,200	4,700	3,100	5,400
12	5	8,200	3,900	6,250	5,000	3,200	5,600
13							
14			Product	Product			
15	Advertising Budget		Unit Cost	Sales Price			
16	$ 30,000.00		$ 15.50	$ 23.00			
17							
18							
19	Number of Ads	Motor Sport	Off-Road	Field & Stream	Dot-Com World	Mad Hacker	Downhill Ski
20	1	0	0	0	0	0	0
21	2	0	0	1	0	0	1
22	3	0	1	0	0	1	0
23	4	1	0	0	0	0	0
24	5	0	0	0	1	0	0
25		1	1	1	1	1	1
26							
27		Motor Sport	Off-Road	Field & Stream	Dot-Com World	Mad Hacker	Downhill Ski
28	Total Ad Cost	$ 8,000	$ 4,800	$ 4,000	$ 7,000	$ 3,000	$ 3,000
29	Unit Sales Generated	8,000	3,500	5,500	5,000	3,000	5,000
30							
31							
32	Revenue	$ 690,000					
33	Advertising Cost	$ 29,800					
34	Production Cost	$ 465,000					
35	Profit	$ 195,200					

(c) What formula should you put in cell B29, the number of products sold as the result of ads placed in *Motor Sport*? Make sure your answer will yield correct results for the other magazines when copied to cells C29:G29.

(d) What formula should you put in cells B32:B35? These cells should contain, respectively, the total revenue from sales resulting from the ad campaign, the total cost of placing the ads, the cost of making the products sold as a result of the campaign, and the total "bottom line" effect of the campaign.

(e) What target cell should you use in the Solver? Should you maximize or minimize it? What are all the constraints you should use in the Solver? Should you check "assume nonnegative"? Should you check "assume linear model"? Suppose you want a solution within 0.5% of the best possible. How would you set the "tolerance" Solver parameter?