16:711:611 Special Topics in Operations Research

Rutgers University
Spring 2009: *Convex Analysis and Optimization*
Professor Jonathan Eckstein

Instructor: Professor Jonathan Eckstein
jeckstei@rci.rutgers.edu
(732) 445-3272 (RUTCOR), (732) 445-0510 (Business School)
http://rutcor.rutgers.edu/~jeckstei

Meeting time: RUTCOR, Wednesdays 1:40-4:40

Text: *Convex Analysis and Optimization*
D. P. Bertsekas, with A. Nedić and A. E. Ozdaglar
Athena Scientific, 2003

Prerequisites

I will expect you to be familiar with the fundamentals of finite-dimensional real analysis, linear algebra, and multivariable differential calculus, as summarized in Section 1.1 of the textbook. I suggest you review this section of the text; if most of the results there are familiar to you, you should have sufficient mathematical background for the course.

Course Content

Convex analysis, the study of convexity and convex bodies, is a field of mathematical analysis that is extremely useful throughout the study of optimization theory and algorithms. This course will cover the basics of finite-dimensional convex analysis and how convex analysis applies to various kinds of optimization problems. Some of the concepts we will study, such as Lagrange multipliers and duality, are also central topics in nonlinear optimization courses; if you take or have taken a course in nonlinear optimization, there will be some unavoidable overlap, but I hope you will get a different, complementary perspective from this course. This course will not cover some other central nonlinear optimization topics, such as gradient, conjugate gradient, Newton, and barrier methods, along with convergence rate analysis – they are very important, but I will leave them to other courses.

Since I have not taught this particular course before, I will try to stick fairly closely to the text for much of the course. I will introduce additional related material along the way, and, time permitting, I may cover some additional topics at the end of the course. In particular, I hope to introduce a different concept of conjugate duality than is introduced in the text. I may distribute additional notes from various sources at some points, including portions of a new, related book Prof. Bertsekas is currently writing (Prof. Bertsekas was my doctoral advisor). Roughly, my plan is to cover one chapter of the textbook every two weeks; however, I hope to cover some chapters a bit faster so as to leave time for related topics not in the text. My ideal planned sequence is:

1. Basic convexity concepts (Sections 1.2-1.5)
    a. Convex sets and functions
    b. Convex and affine hulls
    c. Carathéodory's theorem
    d. Relative interior, closure, continuity, and semicontinuity
    e. Convex cones
    f. Recession cones
2. Separation and optimization (Chapter 2)
    a. Global and local optima
    b. Projection
    c. Recession and existence of optima
    d. Separating and supporting hyperplanes
    e. Bertsekas crossing duality
    f. Saddle and minimax theory
3. Polarity and convex polyhedra (Chapter 3)
    a. Polar cones
    b. The Farkas lemma and Minkowski-Weyl theorem
    c. Extreme points and Carathéodory's theorem revisited
    d. Polyhedral optimization and linear programming
4. Subgradients and locally generated cones (Chapter 4 and additional material)
    a. Subgradients and the subdifferential mapping
    b. Monotone point-to-set operators
    c. Approximate subgradients
    d. Subgradients and directional derivatives
    e. Subgradients and directional derivatives of max functions
    f. Normal and tangent cones
    g. Simple optimality conditions
5. Lagrange multipliers and constraint qualifications (Chapter 5)
6. Simple Lagrangian duality (Sections 6.1-6.5)
7. Rockafellar conjugate duality
    a. Conjugate functions (Section 7.1)
    b. Bifunction conjugate duality (notes from Vanderbei and Çinlar)
8. Convex-analytic computational methods
    a. Subgradient and Lagrangian relaxation methods (Sections 8.1-8.2)
    b. Proximal and augmented Lagrangian methods
    c. Cutting plane and bundle methods (Section 8.3 + additional material)

It may not be possible to cover all these topics, but we will do our best. Note that I will not explicitly cover infinite-dimensional spaces in this class. The key concepts and results are similar in such settings, but there tend to be tricky details that are distracting from the main flow of ideas.

In each class, I will state the key results and explain the most important or illuminating proofs. For some of the material, however, I may omit the proofs during class, and refer you to read the relevant portions of the textbook or supplementary notes.

Additional references

The “bible” of convex analysis is

R. T. Rockafellar. *Convex Analysis*. Princeton University Press, 1970.

Despite being nearly 40 years old, this book is still in print and widely available. It is an exceedingly useful comprehensive reference and very well written, and I encourage anybody working in optimization to purchase a copy. However, it is more of a monograph or reference work than a textbook, containing no explanatory figures and no exercises.

Two textbooks on nonlinear optimization contain material related to the class and may also prove useful:

A. Ruszczyński. *Nonlinear Optimization*. Princeton University Press, 2006.

D. P. Bertsekas. *Nonlinear Programming*. Athena Scientific, 1995/1999.

Both of these texts contain appendices on convex analysis and contain many applications of the theory. I am tentatively planning to use the treatment of bundle methods from the Ruszczyński book (which is also usually the textbook in RUTCOR’s nonlinear optimization class). There are also many other nonlinear optimization books that contain related material.

Assignments and grading

I will hand out a homework assignment every one or two weeks. The assignments will consist largely of mathematical proofs. Two of the assignments, one near the middle of the course, and one at the end, will be longer the rest and serve as take-home exams. Casual collaboration between students will be allowed on the regular assignments, but not on the take-home exam assignments. Your grade will be based on your performance on the assignments, probably 35% for each take-home, and 30% spread among the remaining assignments.

Office hours

I plan to be available 2:00-3:30PM on Tuesdays at RUTCOR, subject to occasional changes and cancellations. I can meet with students at other times by appointment, and I accept e-mail questions at all times.

Staying in touch

I may periodically e-mail the class through Rutgers’ RAMS system to make announcements, such as corrections to assignments. Please make sure that the e-mail address you have on file in Rutgers’ administrative information system is current, and that you check mail at that address regularly. I will also maintain a website of course material and announcements at http://eckstein.rutgers.edu/convex.