26:711:685:03 Special Topics in Management Science: Convex Analysis and Optimization

Rutgers University, Fall 2013
Professor Jonathan Eckstein

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Time: Thursdays, 10:00am-12:50pm
Place: 100 Rockafeller Road (new Business School Building), room 3031
Livingston campus, Piscataway

Note: This class will be essentially identical to the former course 16:711:558, *Convex Analysis and Optimization,* in the New Brunswick graduate school. Although formally in the Newark graduate school, it is being offered on the Livingston campus in Piscataway.

Text: *Convex Analysis and Optimization*
D. P. Bertsekas, with A. Nedić and A. E. Ozdaglar
Athena Scientific, 2003
http://www.athenasc.com/convexity.html

Additional reference material:

R. T. Rockafellar. *Convex Analysis*. Princeton University Press, 1970.
E. Çinlar and R. Vanderbei. *Real and Convex Analysis*. Springer, 2013, or…
R. Vanderbei and E. Çinlar. *Real Analysis for Engineers,* free online book, 2000.
A. Ruszczyński. *Nonlinear Optimization*. Princeton University Press, 2006.
D. P. Bertsekas. *Nonlinear Programming*. Athena Scientific, 1995/1999.

Additional notes will be given out on specific topics.

Prerequisites

I will expect you to be familiar with the fundamentals of finite-dimensional real analysis, linear algebra, and multivariable differential calculus, as summarized in Section 1.1 of the textbook. I suggest you review this section of the text; if most of the results there are familiar to you, you should have sufficient mathematical background for the course.

Course Content

Convex analysis, the study of convexity and convex bodies, is a field of mathematical analysis that is extremely useful throughout the study of optimization theory and algorithms. This course will cover the basics of finite-dimensional convex analysis and how convex analysis applies to various kinds of optimization problems. Some of the concepts we will study, such as Lagrange multipliers and duality, are also central topics in nonlinear optimization courses; if you take or have taken a course in nonlinear optimization, there will be some unavoidable overlap, but I believe you will get a different, complementary perspective from this course. This course will not cover some other central nonlinear optimization topics, such as gradient, conjugate gradient, Newton, and barrier methods, along with convergence rate analysis – they are very important, but I will leave them to other courses.

The course will primarily follow the principle textbook, except for its treatment of conjugate functions and duality, which will instead be patterned after the classic approach of Rockafeller and the treatment in the book by Vanderbei and Çinlar.

Projected week-by-week coverage, subject to change:

1. Introduction, basic convexity concepts -- convex sets, epigraphs, convex functions, closed/lower semicontinuous functions, differentiable convex functions, convex and affine hulls
2. More basic convexity concepts -- cones, Carathéodory's theorem, relative interiors
3. Recession cones, generalized Weierstrass results, local versus global minima, projection
4. Separation and polarity
5. Polyhedral sets and cones, subgradients
6. Monotonicity of subgradients, normal cones, sampling of subdifferential calculus, start constrained optimality
7. Conic approximations, Lagrange multiplier conditions for equality constraints
8. Lagrange multiplier conditions for inequality constraints, start conjugate functions
9. Duality of conjugate functions, simple Fenchel-style duality for optimization problems
10. Examples of Fenchel duality, parametric conjugate (Rockafellar) duality
11. More parametric conjugate duality, start subgradient algorithms
12. Convergence analysis of subgradient algorithms, proximal minimization algorithms
13. Augmented Lagrangian algorithms
14. Augmented Lagrangian algorithms, bundle methods

Note that I will not explicitly cover infinite-dimensional spaces in this class. The key concepts and results are similar in such settings, but there tend to be tricky details that are distracting from the main flow of ideas.

In each class, I will state the key results and explain the most important or illuminating proofs. For some of the material, however, I may omit the proofs during class, and ask you to read the relevant portions of the textbook or supplementary notes.

Additional references

The Rockafellar book mentioned above is generally considered the "bible" of convex analysis. Despite being 40 years old, this book is still in print and widely available. It is an exceedingly useful comprehensive reference and very well written, and I encourage anybody working in optimization to purchase a copy. However, it is more of a monograph or reference work than a textbook, containing no explanatory figures and no exercises. Its treatment of duality uses a difficult "bifunction" notation, and is therefore not very accessible.

The Ruszczyński and Bertesekas textbooks on nonlinear optimization contain material related to the class and may also prove useful. Both of these texts contain appendices on convex analysis and contain many applications of the theory. I am tentatively planning to use the treatment of bundle methods from the Ruszczyński book, which is also usually the textbook in RUTCOR's nonlinear optimization class, so you may also wish to purchase of that book, or at least be prepared to borrow a copy. There are also many other nonlinear optimization books that contain related material.

Assignments and grading

I will hand out a homework assignment every one or two weeks. The assignments will consist predominantly of mathematical proofs. Two of the assignments, one near the middle of the course, and one at the end, will be longer the rest and serve as take-home exams. Casual collaboration between students will be allowed on the regular assignments, but not on the take-home exam assignments. Your grade will be based on your performance on the assignments, probably 35% for each take-home, and 30% spread among the remaining assignments.

Office hours

Will be held weekly, time and place TBA.

Staying in touch

I may periodically e-mail the class through Rutgers' Sakai system to make announcements, such as corrections to assignments. Please make sure that the e-mail address you have on file in Rutgers' administrative information system is current, and that you check mail at that address regularly. I will also maintain a website of course material and announcements at http://eckstein.rutgers.edu/convex. Limited-distribution materials will be placed on Sakai (http://sakai.rutgers.edu).