Nonlinear Optimization

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Homework 1: Elements of Convex Analysis

1. Suppose $f : \mathbb{R}^m \to \overline{\mathbb{R}}$ is convex, A is an $m \times n$ real matrix, and $b \in \mathbb{R}^m$. Show that the function $g(x) = f(Ax + b)$ is convex.

2. A function $f : \mathbb{R}^n \to \overline{\mathbb{R}}$ is called *quasiconvex* if its level sets $M_\beta = \{x \in \mathbb{R}^n \mid f(x) \leq \beta\}$ are convex for all $\beta \in \mathbb{R}$.

   (a) Show that f is quasiconvex if and only if, for all $x^1, x^2 \in \mathbb{R}^n$ and $\lambda \in [0, 1]$, one has $f(\lambda x^1 + (1 - \lambda)x^2) \leq \max\{f(x^1), f(x^2)\}$.

   (b) Give a simple example of a quasiconvex function on $\mathbb{R}$ that is not convex.

   (c) Suppose that $X \subseteq \mathbb{R}^n$ is a convex set and $f : \mathbb{R}^n \to \overline{\mathbb{R}}$ is quasiconvex. Prove that if x^* is a *strict* local minimum of f on X, then it must be the global minimum.

   (d) Give an example of a quasiconvex function on $\mathbb{R}$ that has a non-strict local minimum that is not global.