Business Decision Analytics under Uncertainty  
Fall 2018, Professor Eckstein  
Homework 5  
Due Wednesday, October 24

For each problem below: first, set up the problem for solution by dynamic programming:

- What are the stages in this problem?
- What defines the states within each stage?
- For each stage-state combination, what are the decisions you are choosing between?
- What is the interpretation of the value function \( f(i) \) for this particular problem?

Next, solve the problem by hand using dynamic programming, showing your work. State the optimal solution and its value.

**Q1: Trading Strategies**
You have a sideline business buying and selling vintage jukeboxes. You have room to store up to two jukeboxes and feel certain that the following prices will prevail over the next four months:

<table>
<thead>
<tr>
<th>Month</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sell</td>
<td>$2500</td>
<td>$2800</td>
<td>$2000</td>
<td>$2700</td>
</tr>
<tr>
<td>Buy</td>
<td>$2600</td>
<td>$2900</td>
<td>$2100</td>
<td>$2800</td>
</tr>
</tbody>
</table>

Each month, you have a choice of one of the following three actions:

- Buy: buy one jukebox at the bid price (unless you already have two in storage)
- Hold: do nothing (allowable with any number of jukeboxes in storage, including zero)
- Sell: sell one jukebox at the ask price (unless you do not have any left in storage).

Thus, you may buy or sell at most one jukebox per month. You have a direct holding cost of $100 for each jukebox in storage at the end of each month (for maintenance and cleaning), and want to maximize your profit from trading jukeboxes over the next four months. Assume that you do not have any jukeboxes currently in storage, and that you ascribe a “salvage” value of $2000 to each jukebox left over after the end of the four months (holding cost at the end of month four still applies to any such jukeboxes).

**Q2: An Integer Knapsack Problem**
You have four different kinds of items you can place in a knapsack that can hold up to 7 pounds, as follows:

<table>
<thead>
<tr>
<th>Kind of Item</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight per Item</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Value per item</td>
<td>$22</td>
<td>$20</td>
<td>$33</td>
<td>$5</td>
</tr>
</tbody>
</table>
You can put in as many copies of each type of item into the knapsack as you like, so long as the total weight of the knapsack does not exceed 7 pounds. What loading of the knapsack maximizes its total value?

**Hint:** you can greatly reduce the amount of work by performing a “reachability analysis” to see which states are actually reachable.